pcmac TECHNICAL UPDATE

YOUR QUALITY CALIFORNIA PRODUCERS
Leading the Industry with Improved Precast Products and Systems

PRECAST / PRESTRESSED CONCRETE MANUFACTURERS ASSOCIATION OF CALIFORNIA

100 NORTH BRAND BOULEVARD
SUITE 200
GLENDALE, CALIFORNIA 91203
(818) 247-6177
FAX (818) 240-3041
doug@precastconcrete.org
www.precastconcrete.org

INTEGRAL BENT CAP DESIGN EXAMPLE

BY KOSAL KRISHNAN, Ph.D., P.E. and
AHMAD ABDUL-KARIM, Ph.D., P.E.

This example illustrates the procedure for integral bent cap design in spliced I-girder bridges. The design procedure presented here evolved from successful experimental testing of a scale model of the Florida Bulb Tee girder, at the University of California at San Diego. Holombo et al describe the results of the experimental testing to verify the longitudinal seismic response of precast spliced-girder bridges in [Holombo et. al., 1996].

Given:

Column cross-section:	Circular (ϕ = 6 ft), see **Figure 1**
Column reinforcement:	Vertical- 30 #11 bars (ρ = 1.12 %)
Material properties:	Concrete- f'_c = 4 ksi
	Steel- f_y = 60 ksi
Axial load:	Column top- P = 2,215 k
	Column bottom- P = 2,331 k

Column over-strength moment capacity at the top $M^o_{,top}$ = 11,875 k-ft.
Column over-strength moment capacity at the bottom $M^o_{,bot.}$ = 12,045 k-ft.

Bent Cap Dimension:	7'-0" x 7'-3"
Bent Cap Prestress:	6-19x0.6 in ϕ strands

Required:
Design the integral bent cap to enable transfer of the column plastic moments to the spliced I-girder bridge superstructure.

Solution:
The design procedure involves the following steps:

(1) Calculate principal stresses in the bent cap
(2) Design joint shear reinforcement
(3) Torsion-Shear friction analysis to ascertain the bent cap's ability to transfer column plastic moments to the bridge superstructure.

May, 2001

Step 1: Calculate Principal Stresses in the Bent Cap

Horizontal joint shear force, V_{jh} is given by

$$V_{jh} = \frac{M^o}{h_b} = \frac{11{,}875 \text{ x } 12}{87} = 1{,}637.9 \; kips$$

Where h_b = cap beam section depth = 87 in, and M^o = 11,875 k-ft.

Effective width of cap beam, b_{je} (***Figure 2***)

$$b_{je} = \sqrt{2}\, D \;\; (\; for \; circular \; sections \;)$$

where	D	= column diameter
	b_b	= cap beam width perpendicular to the bridge axis
i.e.,	b_{je}	= 1.414 x 84 = 118.8 in > b_b = 84 in → GOVERNS

Nominal horizontal shear stress level in the joint, v_{jh}

$$v_{jh} = \frac{V_{jh}}{b_{je}\; h_c} = \frac{1637.9}{(84)(72)} = 0.271 \; ksi$$

Calculate f_v = average axial stress in the vertical direction

$$f_v = \frac{P_{col.}}{b_{je}(h_c + 0.5\, h_b)} = \frac{2214.5}{(84)(72 + 0.5(87))} = 0.228 \; ksi$$

$f_h = 0$ (I-girder superstructure- no significant axial stress transferred to cap beam at mid- height of bent cap)

The Principal Tensile Stress, p_t is given by

$$p_t = \frac{f_v + f_h}{2} - \sqrt{\left(\frac{f_v - f_h}{2}\right)^2 + v_{jh}^2}$$

$$i.e., p_t = \frac{0.228 + 0}{2} - \sqrt{\left(\frac{0.228 - 0}{2}\right)^2 + (0.271)^2}$$

$$= -0.180 \; ksi = 2.85\sqrt{f'_{c'}} < 3.5\sqrt{f'_{c'}}$$

According to [Priestley et.al., 1996],

- If principal tension stress $\leq 3.5\sqrt{f'_{c'}}$, no vertical joint reinforcement is needed, and only nominal transverse reinforcement is required.
- If principal tension stress $> 5\sqrt{f'_{c'}}$, all requirements for joint reinforcement must be met.
- If principal tension stress is between $3.5\sqrt{f'_{c'}}$ and $5\sqrt{f'_{c'}}$, linear interpolation between full and nominal requirements for joint reinforcement must be met.

For the purpose of illustration, the cap in this example will be designed for the full joint shear requirements, although the principal tension stress $\leq 3.5\sqrt{f'_{c'}}$, and special vertical joint reinforcement is not needed.

Step 2: Joint Reinforcement Design
The joint design is in accordance with the procedure described by Priestley [Priestley, et. al. 1996]. The assumed joint force transfer mechanism is shown in **Figure 3**. The assumed mechanism (a) reduces congestion by placing joint steel outside column core region, and (b) transfers column tension force to the top of the joint.

Assumptions:

(a) 75% of all column reinforcement is clamped by the main diagonal compression strut D_1 (see **Figure 3**).

(b) Remaining 25% of the total longitudinal column reinforcement at appropriate strain hardening stress T_c is clamped by the diagonal compression struts D_2 and D_3. The vertical components of D_2 and D_3 are assumed to be equal. Joint shear stirrups balance the vertical component of D_2 and help redirect the compression strut C_b into the middle of the joint.

The vertical reinforcement should be placed over a distance of $h_b/2$ from the column face, in accordance with the following equation on each side of the column.

$$A_{jv} = 0.125\, A_{sc} \frac{f^o_{yc}}{f_{yv}}$$

where A_{sc} = total area of longitudinal reinforcement in column section

f^o_{yc} = overstrength stress in column reinforcement including strain hardening & yield overstrength

f_{yv} = yield strength of joint vertical reinforcement

Taking $f^{o}_{yc} = 1.4\ f_{yv}$ for Grade 60 rebar, we have

$$A_{jv} = 0.125(46.8)\frac{1.4\ f_{yv}}{f_{yv}} = 8.2\ in^2$$

Number of # 6 stirrup legs required = A_{jv} /0.44 = 8.2 / 0.44= 18.6, say 20 legs. Provide 10 # 6 (two legs) stirrups on each side of the column face over a distance of h_b / 2 = 87/2 = 43.5 in from the column face.

An additional amount of vertical reinforcement equal to half of this should be placed within the joint confines to help stabilize top beam reinforcement and assist in the transfer of column tension force by bond.

$$A_{vi} = \textit{interior vertical joint stirrup area} = 0.0625\ A_{sc}\frac{f^{o}_{yc}}{f_{yv}}(2)$$

$$i.e.,\ A_{vi} = 0.0625(46.8)\frac{(1.4\ f_{yv})}{f_{yv}} = 4.1\ in^2$$

Number of # 6 stirrup required = A_{vi} /0.44 = 4.1 / 0.44 = 9.3, say 10 legs. Provide 10 # 6 stirrup legs within the column core. As the clamping action occurs at the top of the joint, these stirrups need not extend to the base of the joint. They are extended at least two-thirds of the cap beam depth (2/3 (87) = 58 in). **Figure 4** depicts the location for the placement of vertical joint reinforcement.

<u>**Note**</u>: The reinforcement placed outside the column core, over a length of 0.5 h_b is additional to shear reinforcement required for conventional shear transfer in the beam.

Horizontal hoop reinforcement to carry a force of 0.25 T_c in accordance with the following equations must be provided.

$$\rho_s = \frac{3.3}{D\ f_{yh}\ l_a}\left(\frac{0.09\ A_{sc}\ f^{o}_{yc}\ D}{l_a} - F\right)$$

Assuming F = 0, the equation simplifies to

$$\rho_s = \frac{0.3\ A_{sc}\ f^{o}_{yc}}{l_a^2\ f_{yh}}$$

where D = 72 in = h_c

l_a = assumed length of column anchorage reinforcement in joint = 60 in

$$\therefore \rho_{s,reqd} = \frac{0.3(46.8)(1.4\ f_{yh})}{(60)^2\ f_{yh}} = 0.0055$$

However, a minimum horizontal hoop reinforcement given by the following must be provided.

$$\rho_{s,min.} = \frac{3.5\sqrt{f'_c}}{f_{yh}} = \frac{3.5\sqrt{4000}}{60000} = 0.0037 < \rho_{s,\,reqd.}$$

$$s_{reqd.} = \frac{4\,A_h}{D'\,\rho_{s,reqd.}} = \frac{4\,(0.44)}{(68.25)\,(0.0055)} = 4.7\ in$$

where A_h = area of hoop reinforcement
D' = core diameter of spirally confined column

Provide # 6 stirrups @ 4 in spacing.

Step 3 - Shear Friction Analysis
In the absence of the bottom slab in the case of the bulb tee section, column moments and shears are transferred into the girders completely through torsional mechanisms. Due to the limited length available between the face of the column and the girder, spiral cracks typically associated with torsion cannot fully develop. Therefore, conventional torsion design methodologies that are primarily based on this cracking pattern are not applicable. Instead, the torsional capacity is calculated in terms of the plastic friction model (**Figure 5**).

Assumptions: (a) Shearing stress is assumed constant over the cross section and proportional to the normal force P.
(b) Shear friction contribution of each quadrant is proportional to the area of each quadrant.

The bent cap section is subjected to vertical and horizontal shear forces V_V and V_L, a torque T, and axial clamping force P. It is conceptually divided into four unequal quadrants of areas A_1 to A_4, as shown in **Figure 6**. The direction of shear friction resistance with each of the four quadrants is taken as parallel to the outer edge, and the shear friction stress is taken as $\tau = \mu P/A$, where A is the total section area and μ is the coefficient of friction over the interface. The force in each quadrant F_i is then given by τA_i, where A_i is the area of the quadrant. Equilibrium under external torsion T, longitudinal shear force V_L, and vertical shear force V_V requires that:

$$V_V = F_1 - F_3$$
$$V_L = F_2 - F_4$$
$$T = F_1 x_1 + F_2 y_2 + F_3 x_3 + F_4 y_4$$

The components of the above equations may be solved by trial and error, dividing the section into quadrants until all three equations are satisfied and then checking the implied value of μ.

Alternately, a limit design value of $\mu = 1.4$ could be used and F_1 to F_4 are then selected to satisfy the first two equations shown above. The torque predicted by the third equation must then be checked to ensure that it exceeds the applied torque. The later alternative is utilized in this design solution.

The normal force is calculated assuming a dilation strain of 0.0005 develops on the shear plane.

$$P = P_f + A_s (0.0005) E_s$$

where P_f = prestressing force after all losses (= 0.8 P_i)
A_s = area of reinforcing steel passing through shear plane including prestressing steel
E_s = modulus of elasticity of steel = 29,000 ksi

A_{ps} = 6 (19 x 0.6 in dia strands) = 6 (19 x 0.215) = 24.5 in^2

P_f = (202.5) (0.8) (24.5) = 3,969 kips

A_s = 3 (10) (1.27) = 38.1 in^2 (ignore prestressing steel-conservative)

$\therefore$ P = 3,969 + 38.1 (0.0005) (29000) = 4,521.5 kips

Assume $\mu = 1.4$ (surface of girder in joint region is roughened)

b_b = 84 in ; h_b = 87 in

$$M^0_{i,capbeam} = M_p + V_p \left(\frac{h_b}{2}\right)$$

$$= 11{,}875 + 928.9 \left(\frac{87}{2}\right) = 15{,}242.3\ k-ft$$

$$P_{DLcapbeam} = 2{,}214.5\ kips$$

$$V^0_{i,column} = \left(\frac{12{,}045 + 11{,}875}{25.75}\right) = 928.9\ kips$$

Using a factor of safety of 1.1, we have

$$\therefore\ T_{reqd.} = \frac{M^0_{i,capbeam}(1.1)}{2} = \frac{(15{,}242.3)(1.1)}{2} = 8{,}383.3\ k-ft$$

$$V_L = \frac{V^0_{i,column}(1.1)}{2} = \frac{(928.9)(1.1)}{2} = 510.9\ kips$$

$$P_{DL,capbeam} = \left(\frac{2{,}214.5}{2}\right) = 1{,}107.3\ kips$$

Table 1 detail the torsional shear friction computations to ascertain the ability of the bent cap to transfer the column plastic moment capacity to the bridge superstructure.

Table 1 – Torsional Shear-Friction Computations

HEIGHT = 7.25 ft WIDTH = 7.00 ft AXIAL FORCE = 4521.5 kips FRICTION COEFF. = 1.4		X-COORD = 4.800 ft Y-COORD = 3.000 ft	
SEGMENT	**AREA**	**CENTROID**	**MOMENT**
1	17.400	1.900	33.060
2	14.875	2.208	32.849
3	7.975	2.767	22.064
4	10.500	2.625	27.563
Σ	50.750		115.536
		Capacity	**Demand**
	T (k-ft)	14,410.9	8,383.3
	V_V (k)	1,175.6	1,107.3
	V_L (k)	545.7	510.9

The bridge superstructure moment capacity must also be checked to ensure that the plastic hinges form in the column and not in the superstructure. **Figure 7** depicts the reinforcement details for the integral bent cap.

15.5 REFERENCES

Priestley, M.J.N., Seible, F., and Calvi, G.M., "Seismic Design and Retrofit of Bridges", J. Wiley & Sons, Inc., 1996.

Holombo, J., Priestley, M.J.N., and Seible, F., "Longitudinal Seismic Response of Precast Spliced-Girder Bridges", PCMAC Technical Update, 1996.

LIST OF FIGURES:

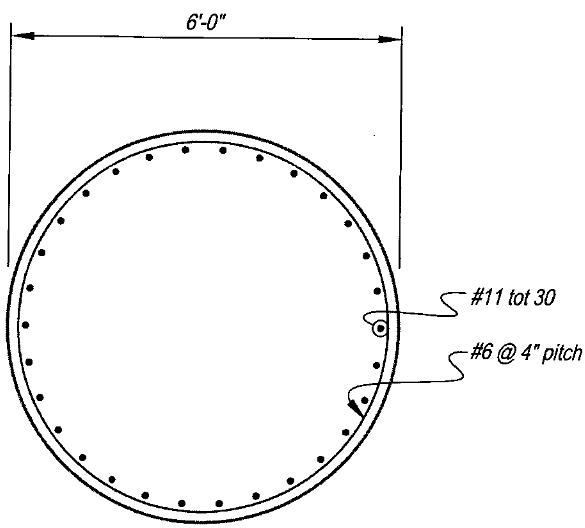

Figure 1 Column Cross-Section

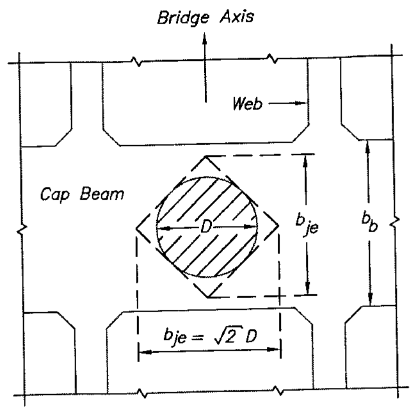

Figure 2 Effective Joint Width for Joint Shear Stress Calculations

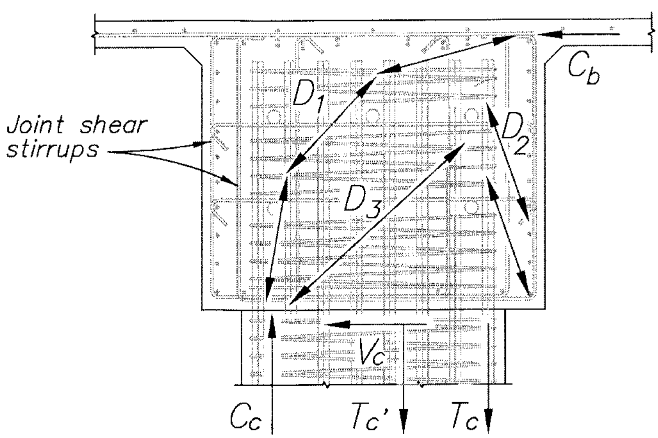

Figure 3 Assumed Mechanism for Joint Force Transfer

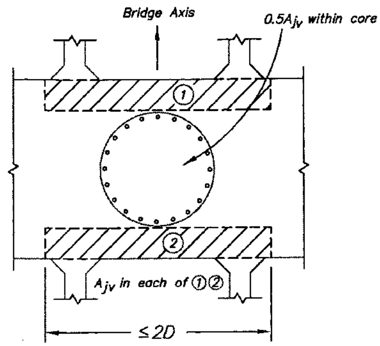

Figure 4 Locations for Vertical Joint Reinforcement

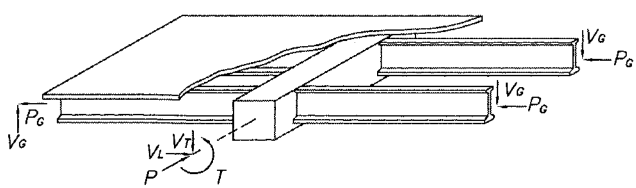

Figure 5 Torsional Shear-Friction Mechanism

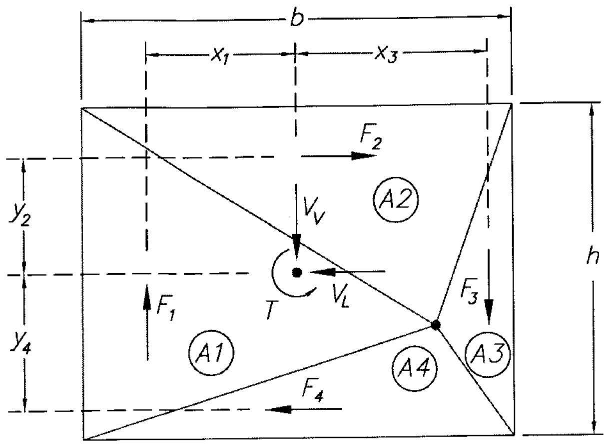

Figure 6 Conceptual Force Diagram for Resisting Torque in Bent Cap

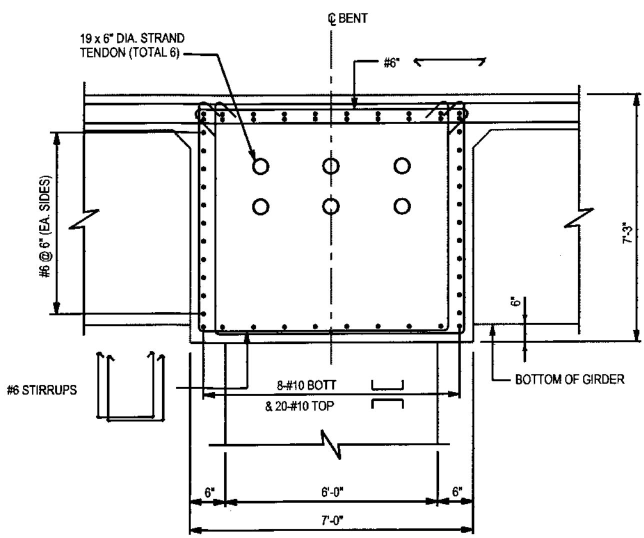

Figure 7 Integral Bent Cap Reinforcement Details